1. Solve the initial value problem

$$y''' = 0 \qquad y(0) = -1 \qquad y'(0) = 0 \qquad y''(0) = 2$$

$$y'' = 2 \qquad y' = 2x \qquad y = x^2 - 1$$

2. Solve the boundary value problem

$$y'' - 10y' + 25y = 0 \qquad y(0) = 1 \qquad y(1) = 0$$

Auxiliary equation

$$m^2 - 10m + 25 = (m-5)^2 = 0$$

$$m_1 = m_2 = 5$$

General Solution

$$c_1 e^{5x} + c_2 x e^{5x}$$

$$c_1 = 1 \qquad c_2 = -1$$

Solution BVP

$$e^{5x} - x e^{5x}$$

3. First verify that x^2 is a solution then find a second solution of
$$x^2y'' + 2xy' - 6y = 0$$

A second solution
$$x^2 \int \frac{e^{-\int \frac{2}{x} dx}}{x^4} dx = \frac{-1}{5x^3}$$

4. Find the orthogonal trajectories of the family of curves

$$y = x + ce^x$$

$$y' = 1 + ce^x$$

$$y - y' = x - 1$$

$$y + \frac{1}{y'} = x - 1$$

$$y' = \frac{1}{x - y - 1}$$

$$\frac{dx}{dy} - x = -y - 1$$

Use an integrating factor to solve this differential equation

Answer:

$$x = y + 2 + ce^y$$

5. Solve

$$(x + 3y)\,dx - (3x + y)\,dy = 0$$

This is a homogeneous d.e. Therefore we use the substitution

$$y = ux$$

$$y' = u + u'x$$

$$\int \frac{3 + u}{1 - u^2}\,du = \int \frac{dx}{x}$$

Answer:

$$(y - x)^2 = c(y + x)$$

6. Solve the first order differential equation

$$y' = \frac{x - y^3 + y^2 \sin x}{3xy^2 + 2y \cos x}$$

$$(x - y^3 + y^2 \sin x)dx - (3xy^2 + 2y \cos x)dy = 0$$

This is exact

Answer:

$$2xy^3 + 2y^2 \cos x = x^2 + c$$

7. Solve

$$\frac{dy}{dx} = \frac{2x+1}{2y} \qquad y(-2) = -1$$

Separable

Answer:

$$y^2 = x^2 + x - 1$$

$$y = -\sqrt{x^2 + x - 1}$$

8. Find a particular solution of the nonhomogeneous equation

$$y'' - 4y' + 3y = 2x$$

and then write down the general solution

$$y_p = \frac{2}{3}x + \frac{8}{9}$$

The general solution of the corresponding homogeneous equation

$$c_1 e^x + c_2 e^{3x}$$

GS

$$c_1 e^x + c_2 e^{3x} + \frac{2}{3}x + \frac{8}{9}$$